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# DETERMINATION OF THE CP VIOLATING PHASE $\gamma$ BY A SUM OVER COMMON DECAY MODES TO $B_s$ AND $\bar{B}_s$

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## Abstract

To help the difficult determination of the angle  $\gamma$  of the unitarity triangle, Aleksan, Dunietz and Kayser have proposed the modes of the type  $K^- D_s^+$ , common to  $B_s$  and  $\bar{B}_s$ . We point out that it is possible to gain in statistics by a sum over all modes with ground state mesons in the final state, i.e.  $K^- D_s^+$ ,  $K^{*-} D_s^+$ ,  $K^- D_s^{*+}$ ,  $K^{*-} D_s^{*+}$ . The delicate point is the relative phase of these different contributions to the dilution factor  $D$  of the time-dependent asymmetry. Each contribution to  $D$  is proportional to a product  $F^{cb} F^{ub} f_{D_s} f_K$  where  $F$  denotes form factors and  $f$  decay constants. Within a definite phase convention, lattice calculations do not show any change in sign when extrapolating to light quarks the form factors and decay constants. Then, we can show that all modes contribute constructively to the dilution factor, except the  $P$ -wave  $K^{*-} D_s^{*+}$ , which is small. Quark model arguments based on wave function overlaps also confirm this stability in sign. By summing over all these modes we find a gain of a factor 6 in statistics relatively to  $K^- D_s^+$ . The dilution factor for the sum  $D_{tot}$  is remarkably stable for theoretical schemes that are not in very strong conflict with data on  $B \rightarrow \psi K(K^*)$  or extrapolated from semileptonic charm form factors, giving  $D_{tot} \gtrsim 0.6$ , always close to  $D(K^- D_s^+)$ .

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Time dependent CP violating asymmetries

$$A(t) \sim D \operatorname{Im} \left[ \frac{q}{p} \frac{\bar{M}}{M} \right] \sin(\Delta M t) \quad (1)$$

depend on the physical quantity

$$\frac{q}{p} \frac{\bar{M}}{M} \quad (2)$$

which is invariant under phase redefinition of the  $|B^0\rangle$ ,  $|\bar{B}^0\rangle$  states.  $M$  and  $\bar{M}$  are the decay amplitudes of  $B^0$  and  $\bar{B}^0$  to some common final state  $|f\rangle$ :

$$M = \langle f | \mathcal{H}_W | B^0 \rangle \quad \bar{M} = \langle f | \mathcal{H}_W | \bar{B}^0 \rangle \quad (3)$$

The mass eigenstates are :

$$|B_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle \quad (4)$$

$D$  is the “dilution factor” that differs from 1 for common final states that are not CP-eigenstates. For the moment, in the expression of the asymmetry (1) we have neglected the possible FSI phases, that we will discuss below. It is obviously important to have a large dilution factor  $D$  since the number of needed pairs  $B_s$ ,  $\bar{B}_s$  to observe a given asymmetry scales like the  $A^{-2}$  or  $D^{-2}$ .

We will adopt Wolfenstein phase convention and parametrization of the CKM matrix [1]. In this convention all CKM matrix elements are real except  $V_{ub}$  and  $V_{td}$  and it is simple to identify which modes will contribute to the determination of the different angles on the unitarity triangle  $\alpha$ ,  $\beta$  and  $\gamma$ . In the Standard Model we have  $|q/p| = 1$  to a very good approximation. In Wolfenstein phase convention  $(q/p)_{B_d}$  is complex since it depends on  $V_{td}$  while  $(q/p)_{B_s}$  is real as it depends on  $V_{ts}$ . On the other hand, the CKM factor of the decay amplitudes is real for  $b \rightarrow c$  transitions while it is complex for  $b \rightarrow u$  transitions. This gives us three different possibilities for a non-vanishing  $\operatorname{Im} \left[ \frac{q}{p} \frac{\bar{M}}{M} \right]$ :

1.  $b \rightarrow u$  transitions of the  $B_d$ - $\bar{B}_d$  system, related to the angle  $\alpha$ ;
2.  $b \rightarrow c$  transitions of the  $B_d$ - $\bar{B}_d$  system, related to  $\beta$ , and
3.  $b \rightarrow u$  transitions of the  $B_s$ - $\bar{B}_s$  system, related to  $\gamma$ .

Of course, this is only true within Wolfenstein approximation up to  $O(\lambda^3)$ , and in the tree approximation, since also Penguin diagrams can complicate the picture and pollute the determination of some angles, mostly  $\alpha$  and  $\gamma$ . Examples of the three types of modes, which are CP eigenstates, are respectively :  $B_d, \bar{B}_d \rightarrow \pi^+\pi^-$  ;  $B_d, \bar{B}_d \rightarrow J/\psi K_S$ , and  $B_s, \bar{B}_s \rightarrow \rho^0 K_S$  [2].

There are several improvements one can think of. First, one can consider modes that can help to cleanly isolate the CP phase we are interested in, avoiding the unwanted phases coming from Penguins. Second, one can try to find non-CP eigenstate common modes to  $B^0$  and  $\bar{B}^0$  that, although not so clean as CP eigenstates, can help to increase the statistics [3]. Third, one can consider sums over some channels or semi-inclusive modes that can increase the statistics if they contribute constructively to the asymmetry [4], [5].

In this paper we will be concerned with the angle  $\gamma$ . Assuming unitarity of the CKM matrix, there are two possible determinations of  $\gamma$ . If  $\alpha$  and  $\beta$  are measured through  $B_d, \bar{B}_d$  decays, then  $\gamma = \pi - \alpha - \beta$  is in principle known. However, there is an independent check, the possible determination outlined above through  $B_s, \bar{B}_s$  decays, for example the CP eigenstate mode  $\rho^0 K_S$  (Fig. 1). This measurement of  $\gamma$  will be complicated by the expected quick  $B_s$ - $\bar{B}_s$  oscillations, that can wash out the CP asymmetry. Moreover, the mode  $\rho^0 K_S$  is expected to have a very small branching ratio ( $10^{-6} - 10^{-7}$ ), since it is not only CKM suppressed by  $V_{ub}$ , as it is necessary, but it is also color suppressed (a further factor 0.2 in amplitude). This mode is also polluted by Penguin diagrams.

Aleksan, Dunietz and Kayser[6] have proposed an alternative way of measuring  $\gamma$ , namely to consider decay modes of the type  $K^- D_s^+$  that are not CP eigenstates but are common to  $B_s$  and  $\bar{B}_s$ , the amplitudes being respectively proportional to  $V_{cs}V_{ub}^*$  and  $V_{cb}V_{us}^*$ , both of order  $\lambda^3$  in terms of the Wolfenstein expansion parameter. This mode is not color suppressed, and one expects a branching ratio of  $O(10^{-4})$ . Moreover, this mode is not polluted by Penguins. Although we have here the drawback of the dilution factor  $D$ , both amplitudes  $B_s, \bar{B}_s \rightarrow K^- D_s^+$  are of the same order and one can expect [6] a large dilution factor for the CP asymmetry.

On the other hand, we have pointed out [4], to help the determination of the angle  $\beta$  (for which the popular CP eigenstate  $\psi K_S$  is usually proposed), to sum over the common decay modes to  $B_d$  and  $\bar{B}_d$  :  $D^+ D^-$  ( $S$ -wave, parity

violating),  $D^+D^{*-} + D^{*+}D^-$  ( $P$ -wave, parity conserving),  $D^{*+}D^{*-}$  ( $S + D$  waves, parity violating;  $P$ -wave, parity conserving). Each individual mode, although Cabibbo-suppressed has a decay rate of the same order as  $\psi K_S$ , which is color suppressed. We have shown, making use of the heavy-quark symmetry and transformation properties of the weak interaction under the operator  $CPe^{i\frac{\pi}{2}\sigma_3^{(c)}}$  where  $\sigma_3^{(c)}$  is the charm quark spin operator collinear to the momentum, that most of the modes (except the exchange diagram for  $D^+D^{*-} + D^{*+}D^-$  and the  $D^{*+}D^{*-}$   $P$ -wave, which have small amplitudes) contribute constructively to the time dependent asymmetry, giving a total dilution factor very close to one. The gain in statistics relatively to  $\psi K_S$  is of the order of 6, although the relative detection efficiency puts  $\psi K_S$  and the sum  $D^+D^- + D^+D^{*-} + D^{*+}D^- + D^{*+}D^{*-}$  on roughly the same footing.

In this paper we would like to examine the same possibility in the sum of the modes  $K^-D_s^+, K^{*-}D_s^+, K^-D_s^{*+}, K^{*-}D_s^{*+}$  (and also their CP-conjugated), that would help the difficult determination of the angle  $\gamma$ . This would be even more interesting than for the determination of  $\beta$ , since we do not have here clean modes like  $\psi K_S$  of the latter case. Of course, we do not have here the simple situation of heavy quark symmetry. As we will see below, assuming factorization, each contribution to the dilution factor  $D$  is proportional to a product  $F^{cb}F^{ub}f_{D_s}f_K$  where  $F$  denotes form factors and  $f$  decay constants. To the heavy-to-heavy meson form factors  $F^{cb}$  and heavy meson decay constants  $f_{D_s}$  we can apply the heavy-quark symmetry [7], but to the heavy-to-light meson form factors  $F^{ub}$  and light meson decay constants  $f_K$  we have weaker rigorous results, like the relation between  $F^{ub}$  and  $F^{uc}$  near zero recoil [8].

The dilution factor  $D$  is a physical quantity, independent of the phase convention of the states. It is convenient to work in a precise phase convention, namely the one defined by the heavy quark symmetry in the case of heavy quarks. For light quarks we can adopt the same convention and exploit the empirical fact that the lattice calculations, that extrapolate from heavy masses (at the charm quark, let us say) to light masses, do not find changes of sign for form factors and decay constants. For example, if within the same phase convention  $f_K$  would have a different sign than  $f_D$ , then lattice calculations would observe the quantity  $f_P/f_D$  to go from 1 to zero and change sign when extrapolating

from  $P = D$  to  $P = K$ . This is not what is observed and we conclude that there is stability in sign of the form factors and decay constants when going from heavy mesons to light mesons. This will be crucial to have a reliable estimation of the dilution factor  $D$  when summing on the different ground state modes. Moreover, quark model calculations also confirm this stability in sign.

Let us consider the final states common to  $B_s$  ( $\bar{b}s$ ) and  $\bar{B}_s$  ( $b\bar{s}$ )

$$\begin{aligned}
|f\rangle &= |K^-(\vec{p})D_s^+(-\vec{p})\rangle \\
&|K^-(\vec{p})D_s^{*+}(\lambda=0, -\vec{p})\rangle \\
&|K^{*-}(\lambda=0, \vec{p})D_s^+(-\vec{p})\rangle \\
&|K^{*-}(\lambda=0, \vec{p})D_s^{*+}(\lambda=0, -\vec{p})\rangle \\
&|K^{*-}(\lambda=\pm, \vec{p})D_s^{*+}(\lambda=\pm, -\vec{p})\rangle
\end{aligned} \tag{5}$$

and their CP conjugate modes :

$$\begin{aligned}
|\bar{f}\rangle &= |K^+(-\vec{p})D_s^-(\vec{p})\rangle \\
&|K^+(-\vec{p})D_s^{*-}(\lambda=0, \vec{p})\rangle \\
&|K^{*+}(\lambda=0, -\vec{p})D_s^-(\vec{p})\rangle \\
&|K^{*+}(\lambda=0, -\vec{p})D_s^{*-}(\lambda=0, \vec{p})\rangle \\
&|K^{*+}(\lambda=\pm, -\vec{p})D_s^{*-}(\lambda=\pm, \vec{p})\rangle
\end{aligned} \tag{6}$$

The spin quantization axis is along the line of flight of the decay products in the  $B_s$  rest frame.

Let us write the relevant asymmetries in our case for a definite type of final state like  $|f\rangle = |K^-(\vec{p})D_s^+(-\vec{p})\rangle$  and its CP conjugate mode  $|\bar{f}\rangle = |K^+(-\vec{p})D_s^-(\vec{p})\rangle$ . Let us call  $M_{1,2}$  the amplitudes  $\bar{M}(f)$ ,  $\bar{M}(\bar{f})$  in eq. (2), respectively for the decays  $\bar{B}_s^0 \rightarrow f$  and  $\bar{B}_s^0 \rightarrow \bar{f}$  with the CKM phases and strong phases factorized out (Fig 2) :

$$\bar{M}(f) = M_1 \quad \bar{M}(\bar{f}) = M_2 e^{i\gamma} e^{-i\delta_s} \quad . \tag{7}$$

The amplitudes for the decays  $B_s^0 \rightarrow \bar{f}$  and  $B_s^0 \rightarrow f$  will write then

$$M(\bar{f}) = -M_1 \quad M(f) = -M_2 e^{i\gamma} e^{-i\delta_s} \quad (8)$$

where, in Wolfenstein parametrization :

$$\frac{V_{ub}^* V_{cs}}{V_{us}^* V_{cb}} \cong \rho + i\eta = r e^{-i\gamma} \quad . \quad (9)$$

The minus sign comes from the phase conventions  $CP|B_s^0\rangle = -|\bar{B}_s^0\rangle$ ,  $CP|f\rangle = |\bar{f}\rangle$ . The phase  $\gamma$  is the angle of the unitarity triangle and  $\delta_s$  is a possible strong phase shift between both amplitudes, that we will discuss below.

Therefore, the quantity defined in (2) will write :

$$\frac{q}{p} \frac{\bar{M}(f)}{M(f)} \cong -\frac{M_1}{M_2} e^{i(\gamma+\delta_s)} \quad (10)$$

since, within Wolfenstein phase convention, for the  $B_s^0$ - $\bar{B}_s^0$  system  $\frac{q}{p}$  is approximately real,  $\frac{q}{p} = 1$ , keeping only the  $t$  quark in the loop.

The time-dependent rates are then given by [2], [6], [9]:

$$\begin{aligned} R(B_{phys}^0(t) \rightarrow f) &\sim (|M_1|^2 + |M_2|^2) [1 - R \cos(\Delta Mt) + D \sin(\gamma + \delta_s) \sin(\Delta Mt)] \\ R(\bar{B}_{phys}^0(t) \rightarrow f) &\sim (|M_1|^2 + |M_2|^2) [1 + R \cos(\Delta Mt) - D \sin(\gamma - \delta_s) \sin(\Delta Mt)] \\ R(B_{phys}^0(t) \rightarrow \bar{f}) &\sim (|M_1|^2 + |M_2|^2) [1 - R \cos(\Delta Mt) - D \sin(\gamma + \delta_s) \sin(\Delta Mt)] \\ R(\bar{B}_{phys}^0(t) \rightarrow \bar{f}) &\sim (|M_1|^2 + |M_2|^2) [1 + R \cos(\Delta Mt) + D \sin(\gamma - \delta_s) \sin(\Delta Mt)] \end{aligned} \quad (11)$$

where

$$R = \frac{|M_1|^2 - |M_2|^2}{|M_1|^2 + |M_2|^2} \quad D = \frac{2M_1 M_2^*}{|M_1|^2 + |M_2|^2} \quad . \quad (12)$$

Note that although the weak and non-trivial strong phases have been factorized,  $M_1$  and  $M_2$ , following the usual conventions, can be pure real or pure imaginary numbers according to the final state and hence the form factors involved, as we will see below. From expressions (11), a useful CP asymmetry that we can consider writes :

$$\left\{ R(B_{phys}^0(t) \rightarrow f) + R(B_{phys}^0(t) \rightarrow \bar{f}) \right\} - \left\{ R(\bar{B}_{phys}^0(t) \rightarrow f) + R(\bar{B}_{phys}^0(t) \rightarrow \bar{f}) \right\} \sim$$

$$\sim D \sin \gamma \cos \delta_s \sin(\Delta Mt) \quad (13)$$

since the term in  $\cos(\Delta Mt)$  cancels when summing over the final states  $f$  and  $\bar{f}$ . In this expression we have two quantities affected by hadronic uncertainties, namely the dilution factor  $D$  and the strong phase  $\delta_s$ . Note that in the asymmetry  $\sin \gamma$  appears and not  $\sin 2\gamma$  like it would be the case if both amplitudes  $M_1, M_2$  were dependent on the same CP phase, like for instance the case of a final CP eigenstate. On the contrary, in our case  $M_1$  and  $M_2$  are not dependent of the same CP phase, as we can see in Fig. 2. It is interesting to note that a curious situation could arise, namely that the asymmetry for the CP eigenstate case  $\rho^0 K_S$  (dependent on  $\sin 2\gamma$ ) could vanish, while the asymmetry for the modes that we consider (dependent on  $\sin \gamma$ ) would be non zero, since the region close to  $\gamma = \pi/2$  is not excluded by the present constraints on the unitarity triangle [2].

We will first compute the dilution factor  $D$  for each single mode and for their sum, and later we will discuss the possible phase  $\delta_s$ .

Let us make a remark concerning CP violation tests using CP eigenstates or non-CP eigenstates. The distinction is not a deep one in the following sense. Starting with non-CP eigenstates like we have done above  $|f\rangle$  and its CP conjugate  $|\bar{f}\rangle = CP|f\rangle$ , we can always define two quantum states, eigenstates of CP :

$$|f_{\pm}\rangle = \frac{1}{\sqrt{2}}|f \pm \bar{f}\rangle \quad (CP = \pm) \quad . \quad (14)$$

In this basis we can also compute the asymmetry, and we find the same result as above when summing over  $|f_{\pm}\rangle$  or over  $|f\rangle, |\bar{f}\rangle$  :

$$\begin{aligned} & \left[ R(B_s^0(t) \rightarrow f_+) + R(B_s^0(t) \rightarrow f_-) \right] - \left[ R(\bar{B}_s^0(t) \rightarrow f_+) + R(\bar{B}_s^0(t) \rightarrow f_-) \right] = \\ & = \left[ R(B_s^0(t) \rightarrow f) + R(B_s^0(t) \rightarrow \bar{f}) \right] - \left[ R(\bar{B}_s^0(t) \rightarrow \bar{f}) + R(\bar{B}_s^0(t) \rightarrow f) \right] \end{aligned} \quad (15)$$

Of course, the two CP eigenstates contribute with different signs to the asymmetry :

$$R(B_s^0(t) \rightarrow f_+) - R(\bar{B}_s^0(t) \rightarrow f_+) = - \left[ R(B_s^0(t) \rightarrow f^-) - R(\bar{B}_s^0(t) \rightarrow f^-) \right] \quad . \quad (16)$$

Working on both bases  $|f_{\pm}\rangle$  and  $|f\rangle, |\bar{f}\rangle$  gives the same final result, if one sums over  $|f_{\pm}\rangle$  or  $|f\rangle, |\bar{f}\rangle$ . However, unlike  $|f\rangle, |\bar{f}\rangle$  the CP eigenstates  $|f_{\pm}\rangle$  are not asymptotic states.

Let us now compute the amplitudes  $M_1$  and  $M_2$ . For the spectator contributions, these amplitudes correspond respectively to the emission of a  $K$  or a  $D_s$  (Figs. 2a and 2b). There are also contributions from the exchange diagrams (Figs. 3a and 3b). The latter are quite small, following the same arguments exposed in ref. 4 (a color factor suppression of 0.2 in amplitude, and a form factor suppression), and we will neglect them.

From the definitions

$$\begin{aligned}\langle P(p)|A_{\mu}|0\rangle &= -if_P p_{\mu} \\ \langle V(p, \lambda)|V_{\mu}|0\rangle &= m_V f_V \epsilon_{\mu}^*(\lambda) \\ \langle P_i|V_{\mu}|P_j\rangle &= \left(p_{\mu}^i + p_{\mu}^j - \frac{m_j^2 - m_i^2}{q^2} q_{\mu}\right) f_+(q^2) + \frac{m_j^2 - m_i^2}{q^2} q_{\mu} f_0(q^2) \\ \langle V_i|A_{\mu}|P_j\rangle &= (m_i + m_j) A_1(q^2) \left(\epsilon_{\mu}^* - \frac{\epsilon^* \cdot q}{q^2} q_{\mu}\right) - A_2(q^2) \frac{\epsilon^* \cdot q}{m_i + m_j} \left(p_{\mu}^i + p_{\mu}^j - \frac{m_j^2 - m_i^2}{q^2} q_{\mu}\right) \\ &\quad + 2m_i A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_{\mu} \\ \langle V_i|V_{\mu}|P_j\rangle &= \frac{2V(q^2)}{m_i + m_j} i \epsilon_{\mu\nu\rho\sigma} p_j^{\nu} p_i^{\rho} \epsilon^{*\sigma} \quad (17)\end{aligned}$$

we find

$$M_1 \left( \bar{B}_s^0 \rightarrow K^-(\vec{p}) D_s^+(-\vec{p}) \right) = -|V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} i f_K (m_B^2 - m_D^2) f_0^{cb}(m_K^2) a_1$$

$$M_2 \left( \bar{B}_s^0 \rightarrow K^+(-\vec{p}) D_s^0(\vec{p}) \right) = -|V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} i f_D (m_B^2 - m_K^2) f_0^{ub}(m_D^2) a_1$$

$$M_1 \left( \bar{B}_s^0 \rightarrow K^{*-}(\lambda=0, \vec{p}) D_s^+(-\vec{p}) \right) = +|V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} 2f_{K^*} m_B f_+^{cb}(m_{K^*}^2) a_1 p$$

$$M_2 \left( \bar{B}_s^0 \rightarrow K^{*+}(\lambda=0, -\vec{p}) D_s^-(\vec{p}) \right) = -|V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} 2f_D m_B A_0^{ub}(m_D^2) a_1 p$$



$$M_1 \left( \bar{B}_s^0 \rightarrow K^-(\vec{p}) D_s^{*+}(\lambda = 0, -\vec{p}) \right) = -|V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} 2f_K m_B A_0^{cb}(m_K^2) a_1 p$$

$$M_2 \left( \bar{B}_s^0 \rightarrow K^+(-\vec{p}) D_s^{*-}(\lambda = 0, \vec{p}) \right) = +|V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} 2f_{D^*} m_B f_+^{ub}(m_{D^*}^2) a_1 p$$

$$M_1 \left( \bar{B}_s^0 \rightarrow K^{*-}(\lambda = 0, \vec{p}) D_s^{*+}(\lambda = 0, -\vec{p}) \right) = |V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} m_{K^*} f_{K^*}$$

$$\left[ (m_B + m_{D^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{cb}(m_{K^*}^2) - \frac{m_B^2}{m_B + m_{D^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{cb}(m_{K^*}^2) \right] a_1$$

$$M_2 \left( \bar{B}_s^0 \rightarrow K^{*+}(\lambda = 0, -\vec{p}) D_s^{*-}(\lambda = 0, \vec{p}) \right) = |V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} m_{D^*} f_{D^*}$$

$$\left[ (m_B + m_{K^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{ub}(m_{D^*}^2) - \frac{m_B^2}{m_B + m_{K^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{ub}(m_{D^*}^2) \right] a_1$$

$$M_1^{pv} \left( \bar{B}_s^0 \rightarrow K^{*-}(\lambda = \pm 1, \vec{p}) D_s^{*+}(\lambda = \pm 1, -\vec{p}) \right) = |V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} m_{K^*} f_{K^*}$$

$$(m_B + m_{K^*}) A_1^{cb}(m_{K^*}^2) a_1$$

$$M_2^{pv} \left( \bar{B}_s^0 \rightarrow K^{*+}(\lambda = \pm 1, -\vec{p}) D_s^{*-}(\lambda = \pm 1, \vec{p}) \right) = |V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} m_{D^*} f_{D^*}$$

$$(m_B + m_{D^*}) A_1^{ub}(m_{D^*}^2) a_1$$

$$M_1^{pc} \left( \bar{B}_s^0 \rightarrow K^{*-}(\lambda = \pm 1, \vec{p}) D_s^{*+}(\lambda = \pm 1, -\vec{p}) \right) = \pm |V_{us}^* V_{cb}| \frac{G}{\sqrt{2}} m_{K^*} f_{K^*}$$

$$\frac{m_B}{m_B + m_{D^*}} 2V^{cb}(m_{K^*}^2) a_1 p$$

$$M_2^{pc} \left( \bar{B}_s^0 \rightarrow K^{*+}(\lambda = \pm 1, -\vec{p}) D_s^{*-}(\lambda = \pm 1, \vec{p}) \right) = \pm |V_{cs}^* V_{ub}| \frac{G}{\sqrt{2}} m_{D^*} f_{D^*}$$

$$\frac{m_B}{m_B + m_{K^*}} 2V^{ub}(m_{D^*}^2) a_1 p \quad (18)$$

In these relations  $p = |\vec{p}|$  is the modulus of the momentum in the center-of-mass, and the weak and strong phases defined in (7)-(8) have been factorized out, except for the trivial but crucial phases that arise from the form factors involved.

We use the phase convention of the heavy quark symmetry. In this phase convention, one has the well-known heavy quark symmetry relations :

$$f_{D^*} = f_D \quad (19)$$

$$\begin{aligned} \frac{2\sqrt{m_B m_D}}{(m_B + m_D)} f^+(q^2) &= \frac{2\sqrt{m_B m_D}}{(m_B + m_D)} \frac{f_0(q^2)}{\left[1 - \frac{q^2}{(m_B + m_D)^2}\right]} = \frac{2\sqrt{m_B m_{D^*}}}{(m_B + m_{D^*})} V(q^2) = \\ &= \frac{2\sqrt{m_B m_{D^*}}}{(m_B + m_{D^*})} A_0(q^2) = \frac{2\sqrt{m_B m_{D^*}}}{(m_B + m_{D^*})} A_2(q^2) = \frac{2\sqrt{m_B m_{D^*}}}{(m_B + m_{D^*})} \frac{A_1(q^2)}{\left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right]} = \xi(v.v') \end{aligned} \quad (20)$$

that, in our problem, apply to the  $b \rightarrow c$  form factors and, within the factorization approximation, to the emission of  $D_s^+$ ,  $D_s^{*+}$ .

For light mesons we will adopt the same phase convention and assume, as suggested by lattice calculations, that the sign of the different form factors and decay constants does not change when extrapolating from heavy to light mesons. This means that we assume the same relative signs as implied by the preceding equations above, e.g.  $f_{K^*}/f_K > 0$ ,  $A_1(q^2)/f_0(q^2) > 0$ , etc., but we do not necessarily assume the heavy quark symmetry ratios (although we could make such hypothesis as a possible Ansatz, see below).

If we consider all the modes  $PP$ ,  $PV$  and  $VV$ , and we neglect the strong phase of each mode, the asymmetry has the same form as in (13), with the dilution factor  $D$  given by

$$D = \frac{2 \sum_i M_1^{(i)} M_2^{(i)*} p(i)}{\sum_i \left[ |M_1^{(i)}|^2 + |M_2^{(i)}|^2 \right] p(i)} \quad (21)$$

where the sum extends over all the modes enumerated above with momenta  $p(i)$ .

To be clear, we will give the different contributions to the numerator,  $M_1(\bar{B}_s^0 \rightarrow f) M_2^*(\bar{B}_s^0 \rightarrow \bar{f})$  with their crucial sign, the denominator being obvious.

$$|f\rangle = |K^-(\vec{p})D_s^+(-\vec{p})\rangle \quad |\bar{f}\rangle = |K^+(-\vec{p})D_s^-(p)\rangle$$

$$M_1 M_2^* = |V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{\sqrt{2}} f_K (m_B^2 - m_D^2) f_0^{cb}(m_K^2) f_D (m_B^2 - m_K^2) f_0^{ub}(m_D^2) (a_1)^2$$

$$|f\rangle = |K^{*-}(\lambda=0, \vec{p})D_s^+(-\vec{p})\rangle \quad |\bar{f}\rangle = -|K^{*+}(\lambda=0, -\vec{p})D_s^-(\vec{p})\rangle$$

$$M_1 M_2^* = |V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{\sqrt{2}} 4 f_{K^*} m_B f_+^{cb}(m_{K^*}^2) f_D m_B A_0^{ub}(m_D^2) p^2 (a_1)^2$$

$$|f\rangle = |K^-(\vec{p})D_s^{*+}(\lambda=0, -\vec{p})\rangle \quad |\bar{f}\rangle = -|K^+(-\vec{p})D_s^{*-}(\lambda=0, \vec{p})\rangle$$

$$M_1 M_2^* = |V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{\sqrt{2}} 4 f_K m_B A_0^{cb}(m_K^2) f_{D^*} m_B f_+^{ub}(m_{D^*}^2) p^2 (a_1)^2$$

$$|f\rangle = |K^{*-}(\lambda=0, \vec{p})D_s^{*+}(\lambda=0, -\vec{p})\rangle \quad |\bar{f}\rangle = |K^{*+}(\lambda=0, -\vec{p})D_s^{*-}(\lambda=0, \vec{p})\rangle$$

$$M_1 M_2^* = |V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{\sqrt{2}} m_{K^*} f_{K^*} m_{D^*} f_{D^*}$$

$$\left[ (m_B + m_{D^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{cb}(m_{K^*}^2) - \frac{m_B^2}{m_B + m_{D^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{cb}(m_{K^*}^2) \right]$$

$$\left[ (m_B + m_{K^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{ub}(m_{D^*}^2) - \frac{m_B^2}{m_B + m_{K^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{ub}(m_{D^*}^2) \right] (a_1)^2$$

$$|f\rangle = |K^{*-}(\lambda=\pm 1, \vec{p})D_s^{*+}(\lambda=\pm 1, -\vec{p})\rangle \quad |\bar{f}\rangle = |K^{*+}(\lambda=\pm 1, -\vec{p})D_s^{*-}(\lambda=\pm 1, \vec{p})\rangle$$

$$M_1^{pv} M_2^{pv*} = |V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{\sqrt{2}} m_{K^*} f_{K^*} m_{D^*} f_{D^*} (m_B + m_{K^*}) A_1^{cb}(m_{K^*}^2)$$

$$(m_B + m_{D^*}) A_1^{ub}(m_{D^*}^2) (a_1)^2$$

$$|f\rangle = |K^{*-}(\lambda=\pm 1, \vec{p})D_s^{*+}(\lambda=\pm 1, -\vec{p})\rangle \quad |\bar{f}\rangle = |K^{*+}(\lambda=\pm 1, -\vec{p})D_s^{*-}(\lambda=\pm 1, \vec{p})\rangle$$

$$M_1^{pc} M_2^{pc*} = -|V_{ub}^* V_{cs} V_{us}^* V_{cb}| \frac{G^2}{2} m_{K^*} f_{K^*} m_{D^*} f_{D^*} 4 \frac{m_B}{m_B + m_{D^*}}$$

$$V^{cb}(m_{K^*}^2) \frac{m_B}{m_B + m_{K^*}} V^{ub}(m_{D^*}^2) p^2 (a_1)^2 \quad . \quad (22)$$

Taking the sign of the  $KD_s$  contribution as reference, we see that only the term with  $K^*D_s^*$  in the  $P$ -wave (parity conserving) contributes with a negative sign. However, we need also to discuss the longitudinal (parity violating)  $K^*D_s^*$  because there can be cancellations in its expression. On the other hand, following our conventions for the  $K^*D$  and  $KD^*$  cases, note the minus sign of the CP conjugate states, that combines with the opposite sign of this case in (18) to give a constructive sign also for these modes. Finally, let us remark that in the limit in which we consider the  $s$  quark as heavy, we recover the relative signs of the spectator diagram of the  $B_d \rightarrow D\bar{D}, \dots$  case studied in ref. 4, as it should.

Owing to the heavy flavor symmetry relations, the heavy quark transition parenthesis is positive :

$$\left[ (m_B + m_{D^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{cb}(m_{K^*}^2) - \frac{m_B^2}{m_B + m_{D^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{cb}(m_{K^*}^2) \right] > 0 . \quad (23)$$

However, we do not have any sound theoretical basis to claim a definite sign for the heavy-to-light parenthesis :

$$\left[ (m_B + m_{K^*}) \left( \frac{p^2 + E_{D^*} E_{K^*}}{m_{D^*} m_{K^*}} \right) A_1^{ub}(m_{D^*}^2) - \frac{m_B^2}{m_B + m_{K^*}} \frac{2p^2}{m_{D^*} m_{K^*}} A_2^{ub}(m_{D^*}^2) \right] . \quad (24)$$

The precise value of the dilution factor (21) is rather uncertain with the present knowledge of heavy-to-light form factors. One must wait for precise data on semileptonic decays  $B \rightarrow \pi(\rho)\ell\nu$  to estimate  $D$ , assuming factorization.

However, one can try to constrain the form factors  $f_+^{ub}, V^{ub}, A_1^{ub}, A_2^{ub}$  from data concerning other heavy-to-light quark transitions. First, we have data on non-leptonic decays like the  $B \rightarrow \psi K, \psi K^*$  transitions. These decays depend on  $b \rightarrow s$  form factors that, assuming factorization, could be related to our case by light flavor  $SU(3)$  symmetry. Second, we have also data on the semileptonic form factors  $D \rightarrow K(K^*)\ell\nu$ , and we can try to extrapolate to the  $b \rightarrow s$  form factors using then the heavy flavor symmetry [8].

Recent data on  $B \rightarrow \psi K, \psi K^*$  (the rates and the ratio  $\Gamma_L/\Gamma_{tot}$  for the latter) are hardly compatible with current models of non-leptonic  $B$  meson

	$R$	$R_L$
Exp.	$1.64 \pm 0.34$	$0.78$ ( <i>ARGUS</i> )[17] $0.80 \pm 0.08 \pm 0.05$ ( <i>CLEO</i> )[17] $0.66 \pm 0.10$ $\begin{smallmatrix} +0.10 \\ -0.08 \end{smallmatrix}$ ( <i>CDF</i> )[18]
BSWI[12]	4.23	0.57
BSWII[14]	1.61	0.36
GISW[15]	1.71	0.06
QCDSR[16]	7.60	0.36

Table 1: *The ratios  $R$  and  $R_L$  defined in the text compared to models of  $B \rightarrow K(K^*)$  form factors.*

decays and with the present knowledge of heavy-to-light form factors. We have performed this analysis in detail in [10] (see also Gourdin, Kamal and Pham [11]). We will make here a brief resumé of our discussion [10] to be able to estimate the dilution factor  $D$  (21), the main object of the present paper. Let us consider the ratio of rates

$$R = \frac{\Gamma(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma(\bar{B}_d^0 \rightarrow \psi K^0)} \quad (25)$$

and the polarization ratio for  $\psi K^{*0}$  (L stands for longitudinal polarization) :

$$R_L = \frac{\Gamma_L(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma_{tot}(\bar{B}_d^0 \rightarrow \psi K^{*0})} \quad (26)$$

The data on  $R$  and  $R_L$  are given in Table 1, together with the predictions of the models of Bauer, Stech and Wirbel on non-leptonic  $B$  decays. BSWI stands for the version of the model in which all form factors are pole-like [12], [13], and BSWII for the latter version [14] where the form factor  $A_1(q^2)$  has a pole shape, and the rest of the form factors, namely  $f_+(q^2)$ ,  $V(q^2)$ ,  $A_2(q^2)$ , have a dipole one, the ratio between both types being a pole, like in the heavy-to-heavy case. We consider only the ratios of rates because the absolute magnitude depends on the effective color factor  $a_2$  (the modes under consideration are of the class II, color suppressed), which is fitted from the data [12], [14]. We present the different values of  $R_L$  from the experiments ARGUS, CLEO and CDF. We see that both descriptions of  $R$  and  $R_L$  are not satisfactory, either  $R$  is too large or  $R_L$  too small by roughly a factor 2.

For completeness, we give the predictions of the form factors of the GISW model [15], that gives a very small value for  $R_L$ , and of QCD sum rules [16], provided to us by P. Ball.

We have also examined [10] the possibility of the extrapolation from the data on  $D \rightarrow K(K^*)$  form factors to the  $B \rightarrow K(K^*)$  form factors using heavy quark symmetry scaling laws [8] in the region of  $q^2$  near  $q_{max}^2$ . Notice that the non-leptonic data give us information at a different kinematic point (at  $q^2 = m_\psi^2$ ) than the data on semileptonic  $D$  decays (at small  $q^2$ ) or the heavy quark limit QCD scaling laws (at  $q_{max}^2$ ). Therefore, going from semileptonic  $D$  decays to non-leptonic  $B \rightarrow \psi K(K^*)$  decays requires a double extrapolation, from  $q^2 = 0$  to  $q_{max}^2$  in  $D \rightarrow K(K^*)$ , and from  $q_{max}^2$  in  $B \rightarrow K(K^*)$  to  $q^2 = m_\psi^2$  in  $B$  non-leptonic decays. These extrapolations are obviously very sensitive to the proposed Ansatz for the  $q^2$  dependence of the form factors, and to the corrections to the heavy quark limit scaling laws near  $q_{max}^2$ .

For the form factors  $D \rightarrow K(K^*)$  at  $q^2 = 0$  we have the world average [19] :

$$f_+^{sc}(0) = 0.77 \pm 0.04$$

$$V^{sc}(0) = 1.16 \pm 0.16$$

$$A_1^{sc}(0) = 0.61 \pm 0.05$$

$$A_2^{sc}(0) = 0.45 \pm 0.09(27)$$

The ratios  $R$  and  $R_L$  depend on the ratios of  $B \rightarrow K(K^*)$  form factors  $f_+^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)$ ,  $V^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)$ ,  $A_2^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)$ . If we impose the scaling laws  $B \rightarrow D$  at  $q_{max}^2$ , plus an Ansatz for the  $q^2$  dependence of the ratios of form factors  $f_+(q^2)/A_1(q^2)$ ,  $V(q^2)/A_1(q^2)$ ,  $A_2(q^2)/A_1(q^2)$ , we can also consider  $R$  and  $R_L$  as functions of the ratios  $f_+^{sc}(0)/A_1^{sc}(0)$ ,  $V^{sc}(0)/A_1^{sc}(0)$ ,  $A_2^{sc}(0)/A_1^{sc}(0)$ . Then we can make a  $q^2$  fit to the data on  $R$ ,  $R_L$  and the ratios of  $D \rightarrow K(K^*)$  form factors at  $q^2 = 0$ . As discussed at length in [10], we can assume several hypothesis concerning the relevant behaviors:

- 1) Scaling laws at  $q_{max}^2$  ;
- 2)  $q^2$  behavior of the ratios  $f_+(q^2)/A_1(q^2)$ ,  $V(q^2)/A_1(q^2)$ ,  $A_2(q^2)/A_1(q^2)$  ;
- 3) absolute value and  $q^2$  dependence of the form factors, let us say  $A_1(q^2)$ .

The cases that we will consider here are :

### 1. Scaling laws at $q_{max}^2$ .

We consider two cases.

**1.1. Asymptotic scaling laws.** There is a constraint from the heavy quark symmetry that relates the form factors, say  $D \rightarrow K$  and  $B \rightarrow K$  near zero recoil  $\vec{q} = 0$  (i.e. at  $q_{max}^2$  for each process) [8] :

$$\begin{aligned} \frac{f_+^{sb}(q_{max}^2)}{f_+^{sc}(q_{max}^2)} &= \frac{V^{sb}(q_{max}^2)}{V^{sc}(q_{max}^2)} = \frac{A_2^{sb}(q_{max}^2)}{A_2^{sc}(q_{max}^2)} = \left(\frac{m_B}{m_D}\right)^{\frac{1}{2}} \\ \frac{A_1^{sb}(q_{max}^2)}{A_1^{sc}(q_{max}^2)} &= \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}}. \end{aligned} \quad (28)$$

**1.2. Softened scaling, i.e. scaling law smoothed by mass corrections.** Motivated by the pole factor in the Isgur-Wise relations (20), that will be shown to appear also in the heavy-light case in a quark model of meson form factors [20], we will assume also another type of extrapolation, namely a constant form factor  $A_1(q^2)$  and a single pole for  $f_+(q^2)$  (at  $q^2 = (m_B + m_K)^2$  or  $(m_D + m_K)^2$  according to the transition), and also for  $V(q^2)$ ,  $A_2(q^2)$  (at  $q^2 = (m_B + m_{K^*})^2$  or  $(m_D + m_{K^*})^2$ ). Interestingly, if one does not neglect the light meson masses, this type of extrapolation leads to a smoothed scaling law that exhibits rather large corrections to the asymptotic scaling (28) :

$$\begin{aligned} \frac{f_+^{sb}(q_{max}^2)}{f_+^{sc}(q_{max}^2)} &= \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left(\frac{m_B + m_K}{m_D + m_K}\right) \\ \frac{V^{sb}(q_{max}^2)}{V^{sc}(q_{max}^2)} &= \frac{A_2^{sb}(q_{max}^2)}{A_2^{sc}(q_{max}^2)} = \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left(\frac{m_B + m_{K^*}}{m_D + m_{K^*}}\right) \\ \frac{A_1^{sb}(q_{max}^2)}{A_1^{sc}(q_{max}^2)} &= \left(\frac{m_B}{m_D}\right)^{\frac{1}{2}} \left(\frac{m_D + m_{K^*}}{m_B + m_{K^*}}\right) \end{aligned} \quad (29)$$

### 2. $q^2$ dependence of the ratios $f_+(q^2)/A_1(q^2)$ , $V(q^2)/A_1(q^2)$ , $A_2(q^2)/A_1(q^2)$ .

Concerning this second question, several models have been proposed: a single pole for all form factors [12] or a single pole for  $A_1(q^2)$  and a dipole for  $f_+(q^2)$ ,  $V(q^2)$ ,  $A_2(q^2)$  [14]. The latter model is inferred from applying the heavy-to-heavy symmetry relations (19) to the heavy-to-light case and assuming a pole form factor for  $A_1(q^2)$ . Both possibilities correspond to the cases:

	$R_L$	$\frac{V^{sc}(0)}{A_1^{sc}(0)}$	$\frac{A_2^{sc}(0)}{A_1^{sc}(0)}$	$\chi^2/DoF$
Exp.	$0.66 \pm 0.14$	$1.90 \pm 0.25$	$0.74 \pm 0.15$	
Extrapolation $D \rightarrow B$				
(I)	0.23	1.66	0.37	16.1
(II)	0.32	1.70	0.48	9.5
(III)	0.39	1.75	0.56	5.7
(IV)	0.49	1.82	0.66	1.9

Table 2: *Fit to the ratio  $R_L$  and to the ratios of form factors  $D \rightarrow K^*$ . Extrapolation corresponds to asymptotic scaling with constant (I), or pole (II) ratios, or to softened scaling with constant (III) or pole (IV) ratios (see the text). For  $R_L$  only CDF data are included. For ARGUS [17] ( $R_L > 0.78$ ) or CLEO [17] ( $R_L = 0.80 \pm 0.08 \pm 0.05$ ) data, one has  $\chi^2/DoF > 9$  in all cases.*

### 2.1. Constant behaviour of the ratio.

### 2.2. Pole behavior of the ratio.

### 3. $q^2$ dependence of $A_1(q^2)$ .

Here we will consider two possibilities : **3.1. Pole behavior.**

**3.2. Constant.** This is a simplifying situation suggested by our quark model [20] that predicts a weak  $q^2$  dependence for this form factor.

In [10] we argue that the case of  $f_+$  is special in the sense that one has a quasi-Goldstone boson in the final state and we begin first with a fit to the ratio  $R_L$  and the ratios of form factors  $V^{sc}(0)/A_1^{sc}(0)$ ,  $A_2^{sc}(0)/A_1^{sc}(0)$ , that concern  $D \rightarrow K^*$  alone, assuming the several extrapolation hypothesis, corresponding to the combinations of scaling and  $q^2$  dependence 1.1, 1.2, 2.1, 2.2 described above, corresponding to asymptotic or softened scaling and to a constant or pole ratio of form factors. In this case we have 3 data and 2 parameters (the ratios of form factors) and hence one degree of freedom:  $DoF = 1$ . The fit is very bad for the CLEO and ARGUS data ( $\chi^2/DoF > 9$ ), but for the CDF data the cases of extrapolation 1.2 (softened scaling) together with 2.1 (constant ratio) or with 2.2 (pole ratios) are qualitatively favored (see Table 2, where we only report the fit to CDF data), especially the former. The best fit corresponds to softened scaling with pole ratios as a function of  $q^2$ .

We then make in ref. [10] an overall  $q^2$  fit to the ratios  $R$ ,  $R_L$  and the ratios of form factors  $f_+^{sc}(0)/A_1^{sc}(0)$ ,  $V^{sc}(0)/A_1^{sc}(0)$ ,  $A_2^{sc}(0)/A_1^{sc}(0)$  (we have



	$R$	$R_L$	$\frac{f_+^{sc}(0)}{A_1^{sc}(0)}$	$\frac{V^{sc}(0)}{A_1^{sc}(0)}$	$\frac{A_2^{sc}(0)}{A_1^{sc}(0)}$	$\chi^2/DoF$
Exp.	$1.64 \pm 0.14$	$0.66 \pm 0.14$	$1.26 \pm 0.12$	$1.90 \pm 0.25$	$0.74 \pm 0.15$	
Extrapolation $D \rightarrow B$						
(I)	1.44	0.23	1.20	1.79	0.34	8.5
(II)	1.60	0.32	1.25	1.72	0.47	4.7
(III)	1.81	0.37	1.32	1.66	0.60	3.2
(IV)	2.15	0.45	1.45	1.62	0.81	4.2

Table 3: *Fit to the ratios  $R$ ,  $R_L$  and to the ratios of form factors  $D \rightarrow K^*$ . Extrapolation corresponds to asymptotic scaling with constant (I), or pole (II) ratio, or to softened scaling with constant (III) or pole (IV) ratio (see the text). For  $R_L$  only CDF data are included. For ARGUS [17] ( $R_L > 0.78$ ) or CLEO [17] ( $R_L = 0.80 \pm 0.08 \pm 0.05$ ) data, one has  $\chi^2/DoF > 8.5$  in all cases.*

now  $DoF = 2$ ) assuming several extrapolation hypothesis. The  $\chi^2/DoF$  turns out to be very bad in all these cases if we stick to the CLEO or ARGUS data for  $R_L$  ( $\chi^2/DoF > 8$ ). We show in Table 3 only the results of the fit in the case of CDF, although in our detailed discussion [10] the CLEO data are included. It is clear from Table 3 that the fits are not good, but we give the whole set as it is useful to see how the different hypothesis compare to the data. Reasonable fits with comparable values of  $\chi^2$  are obtained with softened scaling and constant (III) or pole ratio of form factors (IV), and also asymptotic scaling with pole ratio (II), if we accept the CDF data. We compute the dilution factors for all these cases.

Concerning the CP asymmetries considered in this paper, and in order to be free of a too strict model dependence, we will compute  $D$  in the various theoretical schemes, assuming  $SU(3)$  symmetry. In Table 4 we present the predictions for the dilution factor  $D_{K^-D_s^+}$  of the mode  $K^-D_s^+$ , the total dilution factor  $D_{tot}$ , given by (21), and the statistical gain  $\Gamma_{tot}/\Gamma(K^-D_s^+)$  if we consider the sum over the ground state modes in the asymmetry. For the above discussed extrapolation schemes we retain only those that are not in a too strong conflict with the data on  $B \rightarrow \psi K(K^*)$ , as explained in Table 4. For all relatively reasonable schemes, the sign of the parenthesis (24) is positive, like for the heavy-to-heavy case. The parameters that we have adopted are the recent values  $|V_{cb}| = 0.037$ ,  $|V_{ub}| = 0.003$  (corresponding to  $|V_{ub}|/|V_{cb}| = 0.08$ ),  $\tau_B = 1.49 \cdot 10^{-12}$  sec, and for the effective QCD

	$D_{K^-D_s^+}$	$D_{tot}$	$BR(K^-D_s^+)$	$\Gamma_{tot}/\Gamma(K^-D_s^+)$
BSWI	0.61	0.57	$3.5 \times 10^{-4}$	5.5
BSWII	0.61	0.58	$3.5 \times 10^{-4}$	5.5
GISW	0.06	0.02	$1.1 \times 10^{-4}$	5.8
QCDSR	0.42	0.52	$6.0 \times 10^{-4}$	6.0
Lattice (a)	$0.67 \pm 0.12$	$0.60 \pm 0.15$	$(3.6 \pm 1.9) \times 10^{-4}$	$8.9 \pm 1.3$
Lattice (b)	$0.67 \pm 0.12$	$0.60 \pm 0.15$	$(3.6 \pm 1.9) \times 10^{-4}$	$8.8 \pm 1.4$
Extrapolation $B \rightarrow D$				
(III)	0.51	0.56	$3.3 \times 10^{-4}$	5.8
(IV.1)	0.55	0.51	$3.4 \times 10^{-4}$	5.5
(IV.2)	0.84	0.74	$4.0 \times 10^{-4}$	5.3

Table 4: *Dilution factor for the mode  $K^-D_s^+$  and for the sum of all ground state mesons  $D_{tot}$  and total statistical gain for different theoretical schemes of Tables 1 and 2. Lattice results correspond to a pole shape for  $A_1(q^2)$  and a constant (a) or a pole (b) ratio of form factors. The extrapolations correspond to the values of form factors of the best fits in Table 3 plus the central value  $A_1(0) = 0.61$  from experiment : softened scaling law plus constant ratio between the form factors with pole shape for  $A_1(q^2)$  (III) or softened scaling law plus pole ratio between the form factors with pole shape for  $A_1(q^2)$  (IV.1) or taking  $A_1(q^2)$  to be a constant (IV.2). The case (IV.2) is the favored one on phenomenological grounds (see the text).*

factor of class I decays,  $a_1 = 1.15$ . The Isgur-Wise function at the  $K$  or  $K^*$  mass corresponds roughly to  $\xi(1.5) \simeq 0.60$ . We take the decays constants  $f_{D_s} = 0.23$  GeV and  $f_{D_s^*} = 0.28$  GeV, as suggested by lattice calculations [21]. Although there is not any satisfying scheme, we conclude from our calculations that theoretical schemes that do not give  $R_L$  too small or/and  $R$  too large also give a dilution factor  $D_{tot} \gtrsim 0.6$ , and that we obtain a dilution factor for the sum of all final states that is close to the case  $K^-D_s^+$ , and with a statistical gain of the order of 5-6. Notice that we find somewhat smaller values for the dilution factor  $D(K^-D_s^+)$  than in [6]. This can be traced back to the present smaller value of  $|V_{ub}|/|V_{cb}|$  and the more precise value of the Isgur-Wise function.

A last few comments on the results of Table 4. We should emphasize that the data for  $f_+(q^2)$  seem to favor a single pole [19] at the mass of the corresponding  $1^-$  state and therefore the cases (III) or (IV.2) corresponding to constant or pole ratio of form factors with  $A_1(q^2)$  a pole or a constant are favored from this point of view, while case (IV.1), which assumes a dipole

shape for  $f_+(q^2)$  and a pole for  $A_1(q^2)$  is disfavored. To summarize, from the fits of the Tables 2 and 3, plus the hints from data for the  $q^2$  behavior of  $f_+(q^2)$ , among our assumptions of extrapolation in mass and  $q^2$ , the case (IV.2) seems to be favored, i.e., softened scaling plus pole behavior for  $f_+(q^2)$  and constant behavior for  $A_1(q^2)$ . Lattice results of Table 4 have used the European Lattice Collaboration form factors [21]. The  $1/m_Q$  corrections to the asymptotic scaling law (28) were fitted from the lattice calculation, and for the  $q^2$  dependence of  $A_1(q^2)$  a two parameter fit (pole + constant) has been performed. The  $q^2$  dependence used here differs from the one in [21] where pole dominance was assumed for all the form factors. A detailed discussion is made on this matter in [10]. Notice that the dilution factors are remarkably stable taking into account the uncertainties involved and the variety of theoretical schemes, except for the GISW model [15], that owing to the results of Table 1 presumably cannot apply at large  $q^2$ .

Let us finally briefly comment on the strong phase  $\delta_s$  defined in (7)-(8) which is the strong phase difference (up to the conventional minus sign in (8)) between the amplitudes  $M(f) \equiv M(\bar{B}_s^0 \rightarrow f)$  and  $\bar{M}(f) \equiv M(B_s^0 \rightarrow f)$  i.e. for example for  $|f\rangle = |K^-(\vec{p})D_s^+(-\vec{p})\rangle$  the sign between the two diagrams of Figs. 2a and 2b. This phase appears in the asymmetry (13) and it is a further hadronic uncertainty in the determination of the angle  $\gamma$ . It is clear that rescattering effects of the type  $K^-D_s^+ \rightarrow I \rightarrow K^-D_s^+$  where  $I$  is some intermediate state like  $I = D^0\eta$ , ... can induce strong phases in the amplitudes  $M$  and  $\bar{M}$ . However, these rescattering effects are common to both amplitudes and cannot induce a strong phase shift between them. We can also have first a state with the quantum numbers of  $c\bar{u}$  produced by the weak interaction  $\bar{B}_s^0, B_s^0 \rightarrow I \rightarrow K^-D_s^+$  (with  $I = D^0, D^0\pi^0, \dots$ ). However, any strong phase coming from the absorptive part of these contributions is also common to the amplitudes  $M$  and  $\bar{M}$ , and does not introduce any strong phase shift. The only difference comes from the fact that the initial states are particle or antiparticle, and one could have for example intermediate states, produced by the strong interaction, of the form  $\bar{B}_s^0 \rightarrow B_u^+K^-, \dots B_s^0 \rightarrow B_u^-K^+, \dots$  followed by the weak interaction. However, these intermediate states do not contribute to the absorptive part of the amplitude, since the strong interaction conserves flavor, and these are not allowed decay products. Furthermore, these amplitudes give equal

contributions to both processes  $\bar{B}_s^0 \rightarrow B_u^+ K^-$  and  $B_s^0 \rightarrow B_u^- K^+$  and cannot induce, combined with absorptive processes, a difference of phase. Maybe a possible source of a strong phase shift could come from long distance strong effects that cross the weak interaction from the initial to the final state (short distance effects are supposedly taken into account by the effective color factor  $a_1$ ). It seems to us that the strong phase  $\delta_s$  is either small or it could even be zero, although the matter deserves further investigation.

In conclusion, we find that, summing over all decay modes with ground state mesons in the final state could be very useful for the determination of the CP angle  $\gamma$  of the unitarity triangle, since most of the modes contribute with the same sign to the CP asymmetry. For the sum of the modes  $K^- D_s^+$ ,  $K^{*-} D_s^+$ ,  $K^- D_s^{*+}$ ,  $K^{*-} D_s^{*+}$  we get a dilution factor  $D \gtrsim 0.6$ , of the same order of magnitude as for  $K^- D_s^+$  alone, with a statistical gain of the order of a factor 6. The dilution factor could be even larger owing to the favored extrapolation procedure from  $D$  semileptonic form factors:  $D_{tot} \simeq 0.74$  for softened scaling with a pole for  $f_+(q^2)$  and roughly a constant for  $A_1(q^2)$ . Taking into account the variety of the theoretical schemes, the estimation of the dilution factor  $D$  is surprisingly stable, although one must wait for a more precise knowledge of the heavy-to-light meson form factors to have a firm conclusion. The same exercise can be done for other sums over modes that can be useful in the determination of the angles  $\alpha$  ( $\pi\pi$ ,  $\rho\pi$ ,  $\rho\rho$ ) or  $\beta$  ( $\psi K$ ,  $\psi K^*$ , ...).

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## Figure Captions

1. **Fig. 1** The CP eigenstate mode  $\bar{B}_s \rightarrow \rho^0 K_S$ .
2. **Fig. 2** The two spectator diagram contributions to the decay  $\bar{B}_s \rightarrow K^- D_s^+$ .
3. **Fig. 3** Exchange diagram contributions to the decay  $\bar{B}_s \rightarrow K^- D_s^+$ .

This figure "fig1-1.png" is available in "png" format from:

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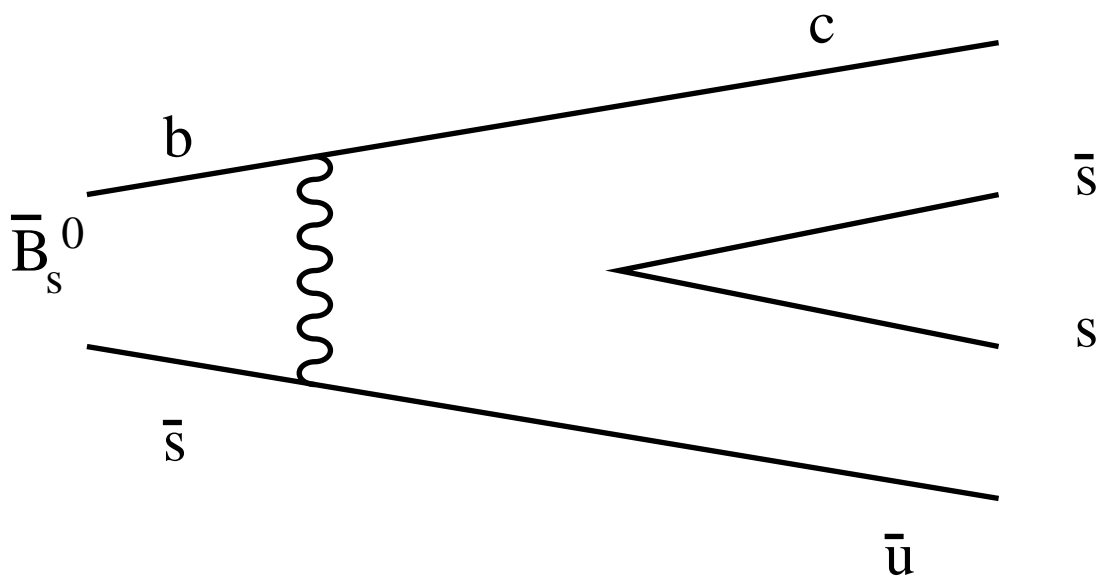


Fig. 3.a

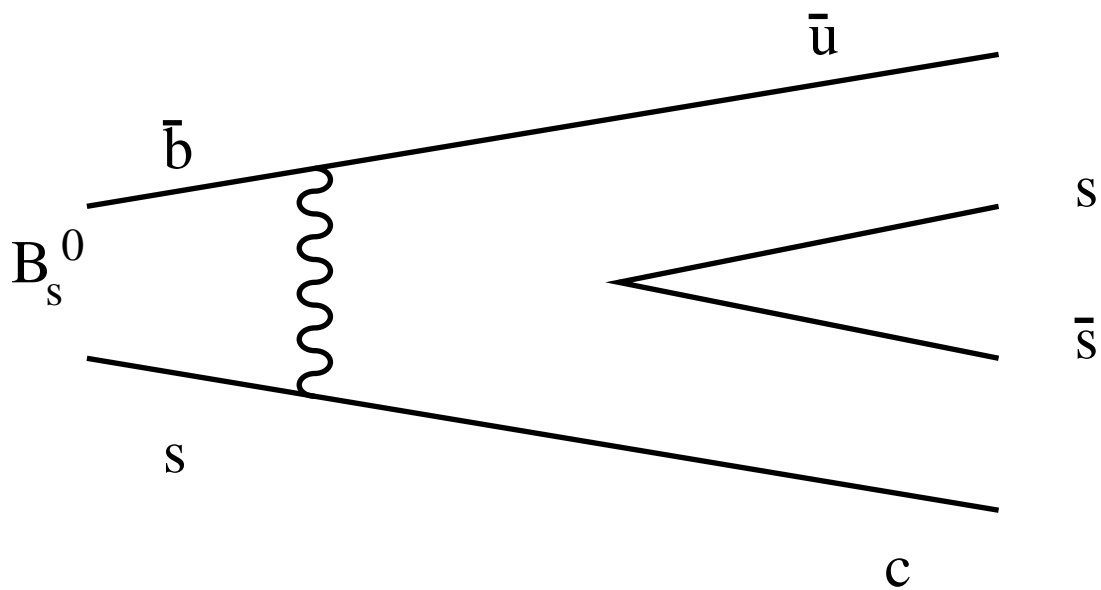


Fig. 3.b

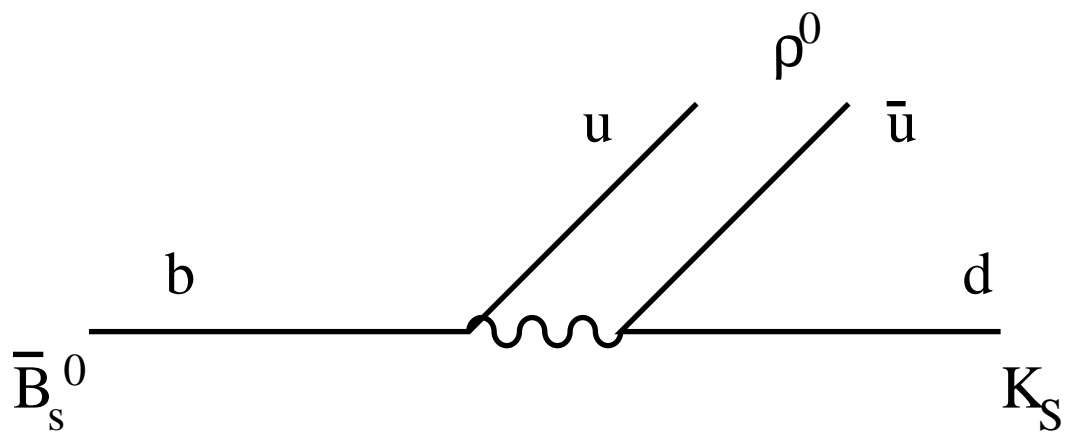


Fig. 1

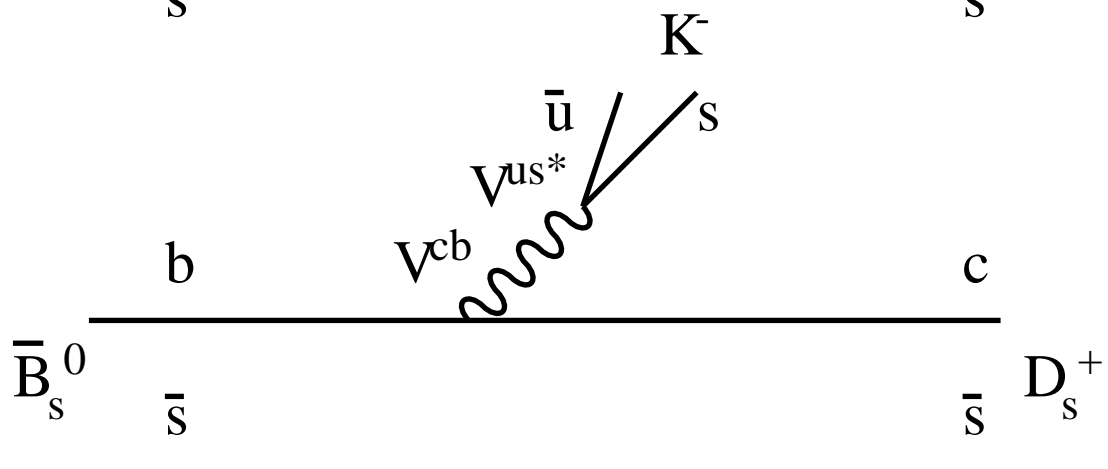


Fig. 2.a

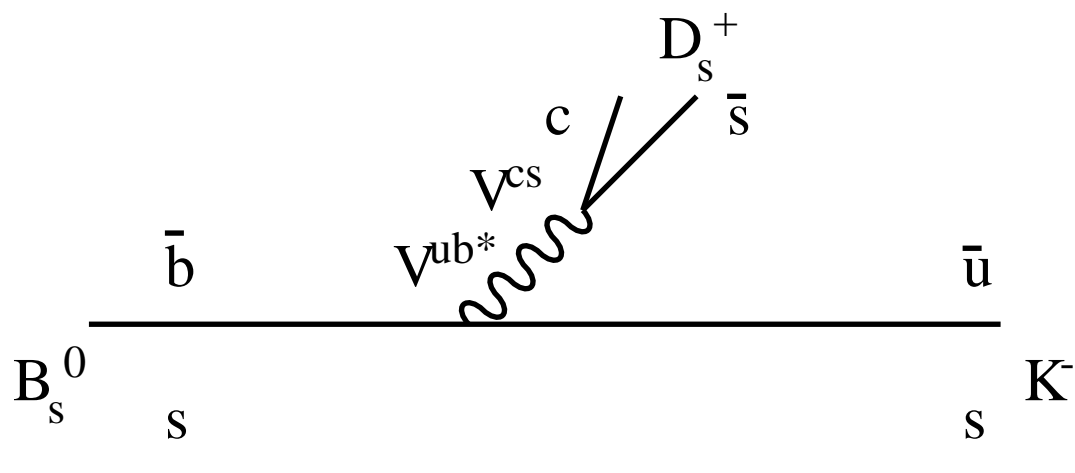


Fig. 2.b